Study of the Nuclear Interaction of Dark Atoms with Matter Nuclei

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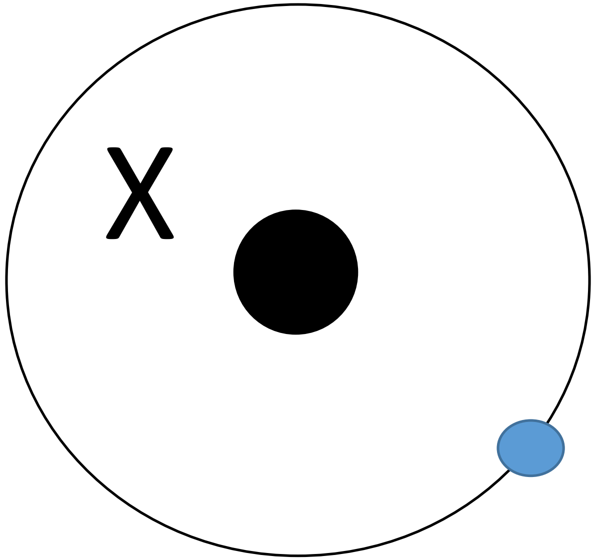
Objectives

1. Determine the reaction rates of Xp(d,γ)X3He using data on the reaction rate of d(p,γ)3He
2. Determine the S-factor of the reaction Xp(d,γ)X3He
3. Calculate the S-factor values for the energy range 0.001MeV – 2MeVTo
4. Determine the dependence of the S-reaction factor on the mass of the dark ion core and its charge

Description of the Dark Atom Model



– heavy, non-baryonic charged particle Z_x

$$Z_x = -2n, n=1,2,3\ldots$$

charged matter particle

Calculation of the Reaction Rate ⟨σv⟩

The reaction rate depends on the S-factor of the reaction as follows:

$$< \sigma v >= \left(\frac{8}{\pi\mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \times \int_0^\infty e^{-2\pi\eta} S(E) e^{-E/kT} dE,$$ [1]

where:
μ — reduced mass of projectile and target,
NA — Avogadro's number,
k — Boltzmann constant,
E — center-of-mass energy.

Determination of the S-factor $S_\chi(E)$ of the reaction **Xp(d,γ)X³He**

The reaction cross section is expressed through the S-factor and the Gamow factor:

$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta},$$

therefore

$$S(E) = E e^{2\pi\eta} \sigma(E).$$

The expression for the reaction cross section is:

$$\sigma(E) = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \cdot \frac{1}{4I(E)} \cdot \frac{\mu}{(2\pi)^2 4\varepsilon_3(E)} \int |M_{fi}^{\mathrm{nucl}}|^2 \, d\Omega_3,$$

where $\varepsilon_3(E) = m_1 + m_2 + E - p_3(E).$

$$I = |\mathbf{p}|(\varepsilon_1 + \varepsilon_2) = p\left(m_1 + m_2 + \frac{p^2}{2\mu}\right) = \sqrt{2\mu E}\,(m_1 + m_2 + E)\,.$$

Determination of the S-factor $S_x(E)$ of the reaction **Xp(d,γ)X³He**

Thus, the final expression for the reaction S-factor was obtained

$$S(E) = E \cdot \frac{2\pi\eta}{1 - e^{-2\pi\eta}} \cdot \frac{1}{4I} \cdot \frac{\mu}{(2\pi)^2 4\varepsilon_3(E)} \int |M_{fi}^{\mathrm{nucl}}|^2 \, d\Omega_3 .$$

We assume that the function $\mathcal{M}$

$$\mathcal{M} = \int |M_{fi}^{\mathrm{nucl}}|^2 \, d\Omega_3 .$$

is assumed to be the same for the reactions Xp(d,γ)X³He and d(p,γ)³He

Determination of the S-factor $S_x(E)$ of the reaction **Xp(d,γ)X³He**

The S-factor Sx(E) for the reaction Xp(d,γ)X³He can be determined from the S-factor of the reaction d(p,γ)³He[2] using the relation

$$S_x(E) = \frac{f_x(E)}{f(E)} \cdot S(E),$$

$$f(E) = \frac{2\pi\eta}{1-e^{-2\pi\eta}} \cdot \frac{1}{4\sqrt{2\mu_{12}E}\,(m_1+m_2+E)} \cdot \frac{\mu}{(2\pi)^2 4\,(m_1+m_2+E-p_3(E))},$$

$$p_3(E) = -m_3 + \sqrt{m_3\,(-m_3+2(m_1+m_2+E))}.$$

$$\eta = Z_1 Z_2 e^2 \cdot \sqrt{\frac{\mu_{12}}{2E}}.$$

- $m_1 = m_p$=938,272 МэВ,
- $m_2 = m_d$=1875,612 МэВ,
- $m_3 = m_{3He}$=2808,391 МэВ.

- $Z_1 = Z_p$=1,
- $Z_2 = Z_d$=1

$$f_{Xp}(E) = \frac{2\pi\eta_x}{1-e^{-2\pi\eta_x}} \cdot \frac{1}{4\sqrt{2\mu_{01}E}\,(m_{Xp}+m_d+E)} \cdot \frac{\mu_{01}}{(2\pi)^2 4\,(m_{Xp}+m_d+E-p_{3x}(E))},$$

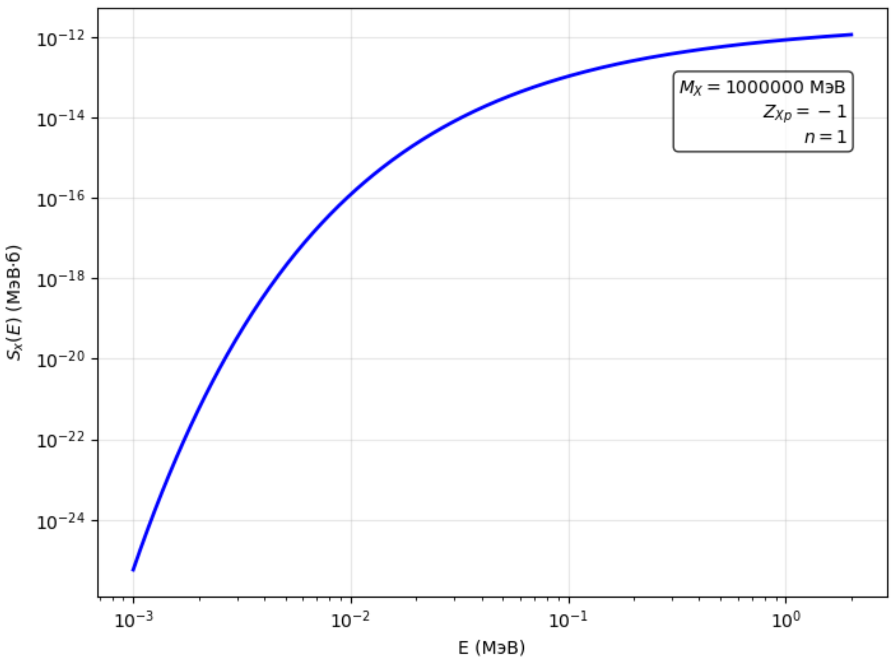
$$p_{3x}(E) = -m_{X3He} + \sqrt{m_{X3He}\,(-m_{3XHe}+2(m_{Xp}+m_d+E))}.$$

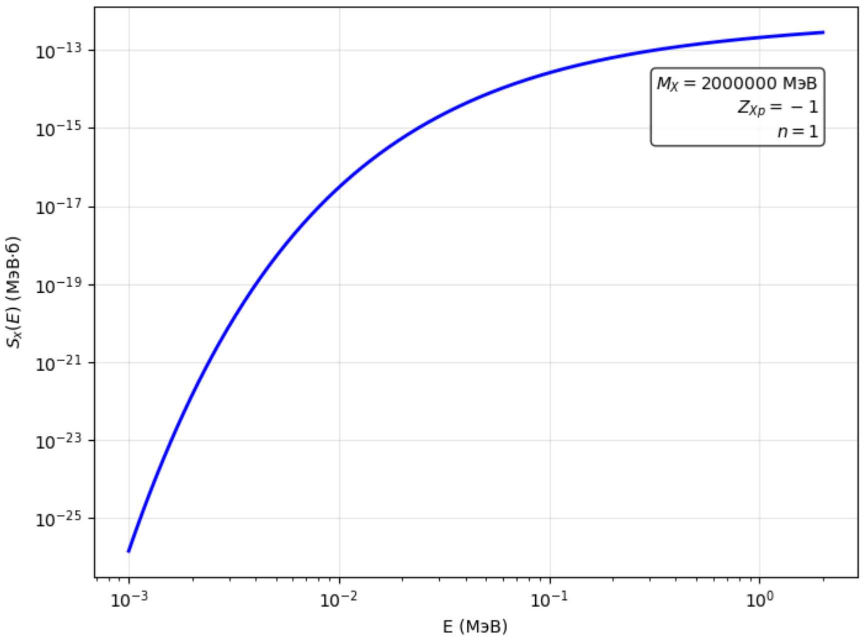
$$\eta_x = Z_{Xp} \cdot Z_d \cdot e^2 \cdot \sqrt{\frac{\mu_{01}}{2E}}.$$

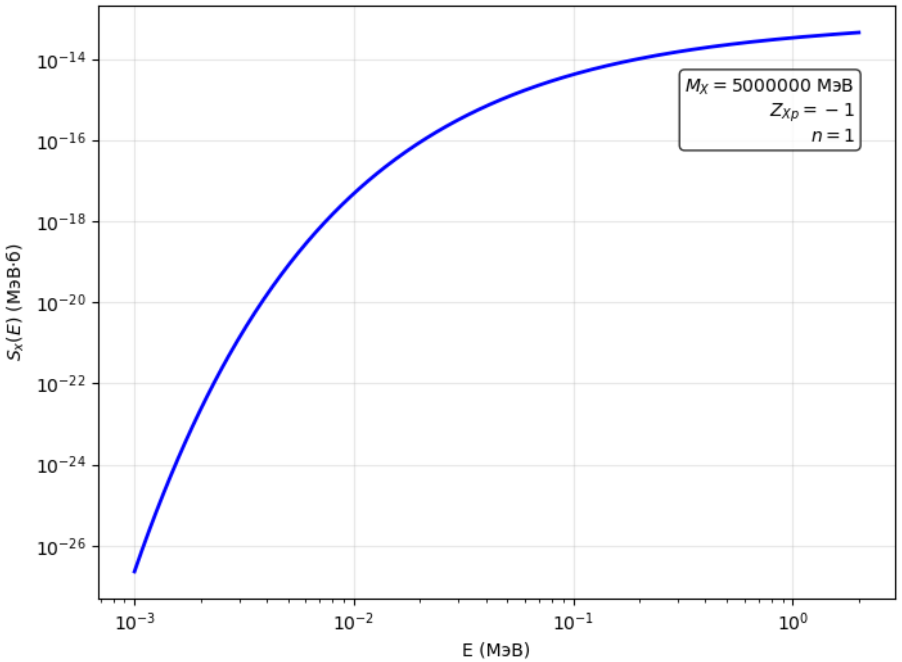
- $m_{Xp} = M_X + m_p - |Q_1|$,
- m_d=1875,612 МэВ,
- $m_{X3He} = M_X + m_{3He} - |Q_2|$,

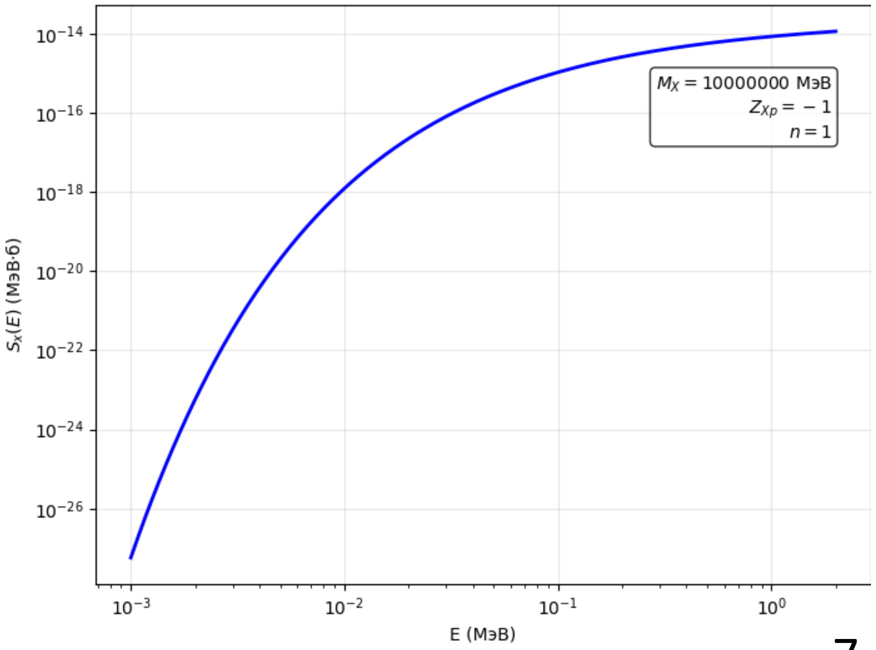
- Z_{Xp} =-2n+1, n=1,2,3...
- Z_d=1,

Dependence of the S-factor on the Dark Ion Nucleus Mass

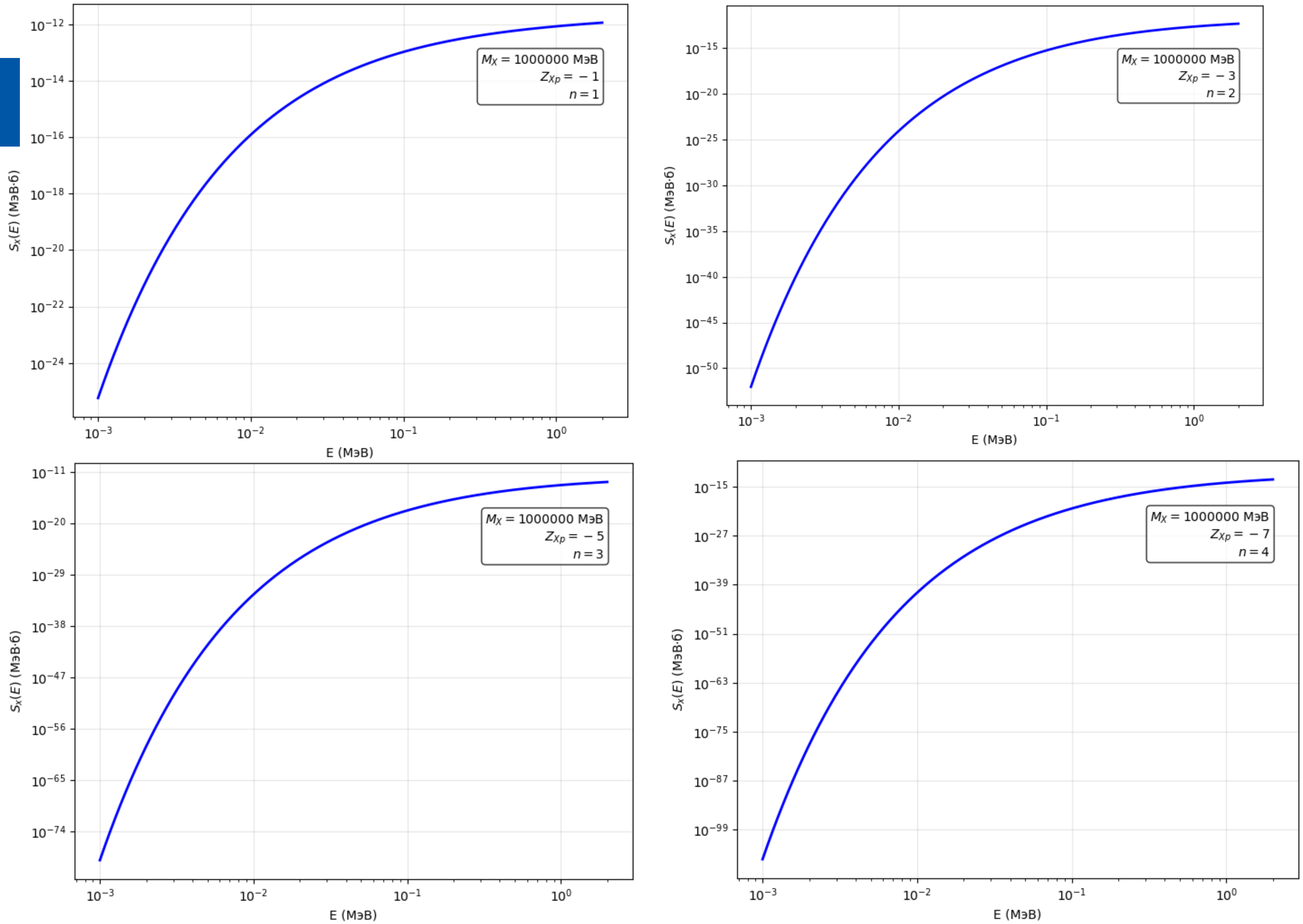








Dependence of the S-factor on the Charge Zx



Conclusion

- The dependence of the S-factor of the Xp(d,γ)X^{3}He reaction on the S-factor of the p(d,γ)^{3}He reaction was determined.
- S-factor values were calculated for the energy range 0.001 MeV – 2 MeV.
- The S-factor depends on the dark nucleus mass and the dark atom charge. Changes in the dark atom charge produce a greater effect on the S-factor than changes in the dark nucleus mass.
- The obtained S-factor values will be used to calculate the Xp(d,γ)X^{3}He reaction rate.

References

[1] Christian Iliadis. Nuclear physics of stars. John Wiley & Sons, 2015.

[2] Joseph Moscoso et al. “Bayesian Estimation of the d (p, γ) 3He Thermonuclear ReactionRate”. In: The Astrophysical Journal 923.1 (2021), p. 49.

[3] Vladimir Berestetskii, Evgeny Lifshitz and Lev Pitaevskii. Theoretical Physics. Vol. 4. Quantum Electrodynamics. LitRes, 2016.

[4] Aleksandr Sergeevich Davydov. Quantum Mechanics. Ripol Klassik, 1968.

Thank you for your attention!

Derivation of the Expression for the Reaction Cross Section σ(E)

The cross section of the 2 → 2 process is described by the general formula:

[3]

$$d\sigma = (2\pi)^4 \delta^{(4)}(P_f - P_i)|M_{fi}|^2 \frac{1}{4I} \prod_a \frac{d^3 p'_a}{(2\pi)^3 2\varepsilon'_a}.$$

Consider a process in which two initial particles 1 and 2 transform into two final particles 3 and 4.
Equation (2) takes the form:

$$d\sigma = (2\pi)^4 \delta^{(4)}(p_3 + p_4 - p_1 - p_2)\,|M_{fi}|^2\,\frac{1}{4I}\,\frac{d^3 p_3}{(2\pi)^3 2\varepsilon_3}\,\frac{d^3 p_4}{(2\pi)^3 2\varepsilon_4},$$

где:

- $\varepsilon_3, \varepsilon_4$ — энергии конечных частиц,
- $I = \sqrt{(p_1 p_2)^2 - m_1^2 m_2^2}$.

Derivation of the Expression for the Reaction Cross Section σ(E)

The four-dimensional delta function decomposes into the product of delta functions in energy and momentum:

$$\delta^{(4)}(p_3 + p_4 - p_1 - p_2) = \delta(\varepsilon_3 + \varepsilon_4 - \varepsilon_1 - \varepsilon_2) \cdot \delta^{(3)}(\mathbf{p}_3 + \mathbf{p}_4 - \mathbf{p}_1 - \mathbf{p}_2).$$

Снятие $\delta^{(3)}$

Интегрируем выражение для $d\sigma$ по d^3p_4:

$$\int d^3p_4\, \delta^{(3)}(\mathbf{p}_3 + \mathbf{p}_4)\frac{1}{2\varepsilon_4} = \frac{1}{2\varepsilon_4(\mathbf{p}_3)},$$

после интегрирования $\mathbf{p}_4 = -\mathbf{p}_3$, а ε_4 становится функцией $p_3 = |\mathbf{p}_3|$.

После этого выражение принимает вид:

$$d\sigma = \delta\big(\varepsilon_3(p_3) + \varepsilon_4(p_3) - \varepsilon_1 - \varepsilon_2\big)\, |M_{fi}|^2\, \frac{1}{4I}\, \frac{d^3p_3}{(2\pi)^2\, 4\, \varepsilon_3(p_3)\, \varepsilon_4(p_3)}.$$

$$d^3p_3 = p_3^2\, dp_3\, d\Omega_3, \quad d\Omega_3 = \sin\theta_3\, d\theta_3\, d\varphi_3.$$

Таким образом:

$$d\sigma = \delta\big(\varepsilon_3(p_3) + \varepsilon_4(p_3) - \varepsilon_1 - \varepsilon_2\big)\, |M_{fi}|^2\, \frac{1}{4I}\, \frac{p_3^2\, dp_3\, d\Omega_3}{(2\pi)^2\, 4\, \varepsilon_3(p_3)\, \varepsilon_4(p_3)}.$$

Вывод выражения для сечения реакции σ(E)

For the process 1+2 → 3+γ in the nonrelativistic limit in the center-of-mass frame:

$$d\sigma = \delta\big(\varepsilon_3(p_3) + p_3 - \varepsilon_1(p) - \varepsilon_2(p)\big)\,|M_{fi}|^2\,\frac{1}{4I}\,\frac{p_3^2\,dp_3\,d\Omega_3}{(2\pi)^2\,4\,\varepsilon_3(p_3)\,p_3},$$

Заметим, что

$$p_3 dp_3 = \frac{1}{2}d(p_3^2) = \frac{2\mu}{2}d\left(\frac{p_3^2}{2\mu}\right) = \mu\,dE$$

Сняв δ(ε3(p3)+p3 −ε1(p)−ε2(p)) получим

$$d\sigma = |M_{fi}|^2\frac{1}{4I(E)}\frac{\mu\,d\Omega_3}{(2\pi)^2\,4\,(m_1 + m_2 + E - p_3(E))},$$

Derivation of the Expression for the Reaction Cross Section σ(E)

Consider the matrix element:

$$|M_{fi}|^2 = |M_{fi}^{\mathrm{nucl}}|^2 \cdot |\psi(0)|^2,$$

where $|\psi(0)|^2$ is the square of the relative motion wave function at the origin.

$$|\psi(0)|^2 = \frac{2\pi\eta}{e^{2\pi\eta} - 1},$$

and the Sommerfeld parameter η is defined as

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar} \cdot \frac{\mu}{p}, \quad p = \sqrt{2\mu E}.$$

Then

$$\sigma(E) = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \cdot \frac{1}{4I(E)} \cdot \frac{\mu}{(2\pi)^2 4\varepsilon_3(E)} \int |M_{fi}^{\mathrm{nucl}}|^2 \, d\Omega_3.$$

Вывод выражения для расчета S-фактора $S_x(E)$ реакции **Xp(d,γ)X³He**

The S-factor is related to the energy and reaction cross section as follows:

$$S(E) = Ee^{2\pi\eta}\sigma(E).$$

Substituting the expression for σ(E), we obtain:

$$S(E) = E \cdot \frac{2\pi\eta}{1 - e^{-2\pi\eta}} \cdot \frac{1}{4I} \cdot \frac{\mu}{(2\pi)^2 4\varepsilon_3(E)} \int |M_{fi}^{\mathrm{nucl}}|^2 \, d\Omega_3.$$